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P
Your Roll No.....

Sr. No. of Question Paper : 4810

E

Unique Paper Code : 32351403

Name of the Paper : Ring Theory & Linear
Algebra – I

Name of the Course : B.Sc. [Hons.] Mathematics
CBCS (LOCF)

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.

P.T.O.

1. (a) Find all the zero divisors and units in $\mathbb{Z}_3 \oplus \mathbb{Z}_6$.

(6)

- (b) Prove that characteristic of an integral domain is

0 or prime number p .

(6)

- (c) State and prove the Subring test

(6)

2. (a) Let R be a commutative ring with unity and

let A be an ideal of R then prove that R/A

is a field if and only if A is a maximal ideal of

R .

(6)

- (b) Let A and B are two ideals of a commutative

ring R with unity and $A+B=R$ then show that

$$A \cap B = AB.$$

(6)

- (c) If an ideal I of a ring R contains a unit then show that $I=R$. Hence prove that the only ideals of a field F are $\{0\}$ and F itself. (6)

3. (a) Find all ring homomorphism from $\mathbb{Z}_6$ to $\mathbb{Z}_{15}$. (6.5)

- (b) Let $R = \left\{ \begin{bmatrix} a & b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Z} \right\}$ and Φ be the mapping

that takes $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ to $a-b$. Show that

- (i) Φ is a ring homomorphism.

- (ii) Determine $\text{Ker } \Phi$.

- (iii) Show that $R/\text{Ker } \Phi$ is isomorphic to $\mathbb{Z}$.

(6.5)

P.T.O.

- (c) Using homomorphism, prove that an integer n with decimal representation $a_k a_{k-1} \dots a_0$ is divisible by 9 iff $a_k + a_{k-1} + \dots + a_0$ is divisible by 9.

(6.5)

4. (a) Let $V(\mathbb{R})$ be the vector space of all real valued function over $\mathbb{R}$.

$$\text{Let } V_e = \{f \in V \mid f(x) = f(-x) \quad \forall x \in \mathbb{R}\}$$

$$\text{and } V_o = \{f \in V \mid f(-x) = -f(x) \quad \forall x \in \mathbb{R}\}$$

Prove that V_e and V_o are subspaces of V and

$$V = V_e \oplus V_o. \quad (6)$$

(b) Let $V(F)$ be a vector space and let $S_1 \subseteq S_2 \subseteq V$.

Prove that

(i) If S_1 is linearly dependent then S_2 is linearly dependent

(ii) If S_2 is linearly independent then S_1 is linearly independent (6)

(c) Show that $S = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\}$ forms a basis for $M_{2 \times 2}(\mathbb{R})$. (6)

5. (a) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation defined by

$$T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3).$$

Find Null space and Range space of T and verify Dimension Theorem. (6.5)

(b) Define $T: M_{2 \times 2}(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ by $T \begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a + b) +$

$$(2d)x + bx^2$$

Let $\beta = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ and

$\gamma = \{1, x, x^2\}$ be basis of $M_{2 \times 2}(\mathbb{R})$ and $P_2(\mathbb{R})$ respectively. Compute $[T]_{\beta}^{\gamma}$. (6.5)

(c) Let V and W be vector spaces over F , and suppose

that $\{v_1, v_2, \dots, v_n\}$ be a basis for V . For $w_1, w_2, \dots, w_n$

in W . Prove that there exists exactly one linear

transformation $T: V \rightarrow W$ such that $T(v_i) = w_i$ for

$i = 1, 2, \dots, n$. (6.5)

6. (a) Let T be the linear operator on $\mathbb{R}^2$ define by

$$T \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a + b \\ a - 3b \end{pmatrix}$$

Let β be the standard ordered basis for $\mathbb{R}^2$ and let

$$\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}.$$

Find $[T]_{\beta'}$.

(6.5)

- (b) Let V and W be finite dimensional vector spaces with ordered basis β and γ respectively. Let $T: V \rightarrow W$ be linear. Then T is invertible if and only if $[T]_{\gamma}^{\beta}$ is invertible.

$$\text{Furthermore, } [T^{-1}]_{\gamma}^{\beta} = ([T]_{\gamma}^{\beta})^{-1}. \quad (6.5)$$

P.T.O.

(c) Let V , W and Z be finite dimensional vector spaces with ordered basis α , β , γ respectively. Let $T: V \rightarrow W$ and $U: W \rightarrow Z$ be linear transformations.

$$\text{Then } [UT]_{\alpha}^{\gamma} = [U]_{\beta}^{\gamma} [T]_{\alpha}^{\beta}. \quad (6.5)$$

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